NSW GOVERNMENT
Education & Communities
Public Schools NSW

Developing numerate students

The Mathematics block

A guide for Early Action for Success schools

> Numeracy has come to be an indispensable tool to the understanding and mastery of all phenomena
>
> (Crowther Report, Par. 401)

What do we mean by numeracy?

Numeracy is the ability to effectively use the mathematics required to meet the general demands of life at home and at work, and for participation in community and civic life. As a newer term than its twin 'literacy', numeracy has been used interchangeably with terms such as 'mathematical literacy' and 'quantitative literacy'. Regardless of which term is used, numeracy involves using some mathematical ideas effectively to make sense of the world.

"…there is the need in the modern world to think quantitatively, to realise how far our problems are problems of degree even when they appear as problems of kind." (par. 401)

The significance of mathematics learning

Each individual's interaction with the world draws on reasoning with number, measurement, probability, data and spatial sense. This reasoning is used in estimating and measuring time and distance, as well as likelihood. Although most people are not consciously aware of it, *quantitative reasoning is required to cross a road, just as it is required to construct a road*.

One hundred years ago, H. G. Wells predicted that "statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write". The time foreshadowed by Wells has arrived.

Developing an understanding of mathematics that enables students to confidently use some mathematics in their day-to-day lives is critical to being numerate.

The importance of early mathematics learning

Children start school reasoning with widely varied mathematics skills. For example, when children start school in NSW about 11% cannot consistently count from one to ten. However, more than 15% can count to at least thirty and can state the number that immediately follows another, such as twenty-three, without starting the count from one.

Recent research is providing clear messages that basic mathematics knowledge at school entry, such as understanding how numbers represent specific magnitudes, is a stronger predictor of later success in life than either basic literacy skills or social behaviour (Duncan et al., 2007; Jordan, 2010). Research data from six longitudinal studies involving over 36 000 preschoolers were analysed to determine factors important for school success in children starting school. The analysis revealed that early behavior problems and social skills were not associated with later reading and mathematics achievement. Rather, early academic skills appear to be the strongest predictor of subsequent scholastic success – early mathematics skills more so than early reading skills.

Controlling for IQ, family income, gender, temperament, type of previous educational experience, and whether children came from one or two parent families, the study found that having early mathematics concepts on school entry was the strongest predictor of future academic success.

The differences between mathematics and reading

Experiences in the teaching of reading do not simply transfer to the teaching of mathematics. Unlike assessments of reading (Figure 1), as students progress from Year 2 to Year 7 in mathematics (Figure 2), the tail of the distribution tends to grow.

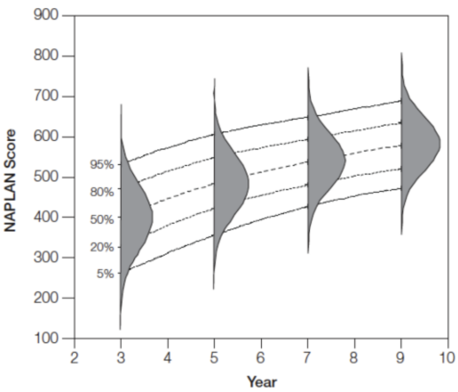

Figure 1. *Distributions of students' reading abilities (Years 3, 5, 7 and 9, Australia, 2008)* [Masters, 2013, p. 13]

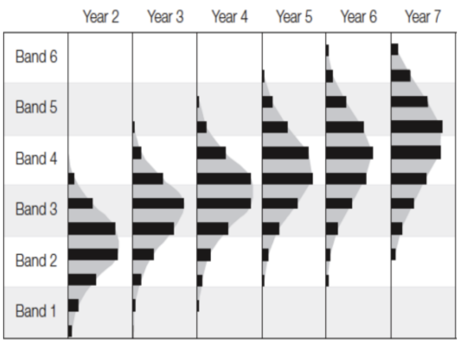

Figure 2 *Distributions of students' mathematics achievements (Years 2–7, USA, 2003)* [Masters, 2013, p. 14]

If you fall behind in mathematics, you are more likely to remain behind than if you fall behind in reading. ***Strong early mathematics teaching is critical to the academic performance of students.***

Mathematics education is, in part, education in language. Students learn to speak, read and write the language of mathematics in order to communicate mathematical ideas. They learn to talk about their own thinking: *I added by counting on my fingers.* In addition to learning letter/sound combinations in reading, they struggle with the formal symbol system of mathematics, which has its own syntax. This special form of written symbolism is perhaps the hardest form of language for many students to learn.

It isn't possible to 'sound out' mathematics symbols and even the traditional left-to-right direction of English text is not always applied. Reading 8 ÷ 4 does not follow the same direction as $4\overline{)8}$. Moreover, the symbols stand for complete words or phrases such as 'divided by', rather than sounds.

Linking teaching and learning

Beliefs about being numerate
One view of what it means to be numerate is 'to know a rule and to be able to use it'. The limitation of this view of being numerate is that potentially every new situation might require a new rule. This belief was found to be associated with ineffective teaching of numeracy (Askew et al., 1997).

Effective teachers of numeracy emphasised the need for students to have a rich network of connections between different mathematical ideas and being able to select and use strategies, which are both efficient and effective. To achieve this outcome they used teaching approaches that:

- ensured that all students were being challenged and stretched, not just those who were more able
- built upon pupils' own mental strategies for calculating, and helped them to become more efficient. (p. 3)

Personalising learning in mathematics

The most important single factor influencing learning is what the learner already knows.
(Ausubel, 1968, p. vi)

It follows that if what the learner already knows is the most important single factor influencing learning, the teacher needs to be able to ascertain this for each and every student in his or her class. That is, teachers need to:

- determine what the learner already knows with respect to the planned learning outcomes
- provide opportunities for students to make their thinking visible
- continuously monitor and assess students' thinking and
- provide or manage feedback and opportunities for practice and reflection.

Students (both those who have been successful in a system that rewards safe answers and those who are accustomed to failure) are often unwilling to confront challenges and take the risks associated with making their thinking visible.

Using the numeracy continuum

The numeracy continuum is a synthesis of many research studies into how different aspects of mathematics develop. It describes the progression of knowledge development as well as the transformations that take place in the strategies students use to solve problems. In particular it outlines how students progress to using increasingly sophisticated strategies for quantifying.

The aspects of the numeracy continuum are interdependent. Children's object counting provides the foundations for work across the four operations in the development of symbolic arithmetic.

Students learn to link quantity with spoken, heard, read and written forms of number. Moreover, the sequence of number words plays an important role as students learn that the last number stated is more than a description of the final item, but also corresponds to the total.

Students also learn to reason about unknown quantities, linking addition and subtraction as well as multiplication and division. As they become familiar with multiples of ten they develop the basis of understanding the power of the place value system.

The numeracy continuum provides a strong basis for assessment of learning as well as assessment for learning. With the clear progression of students' arithmetical strategies, it provides direction for teaching ('where to next') as well as the tools for tracking and monitoring progress to personalise learning and set progress targets.

How long is a mathematics block?

Daily mathematics blocks, typically of about one hour, have been used to provide time for ongoing assessment, differentiated instruction, and student-based, open-ended activities used to encourage higher-order thinking.

The main value of a mathematics block is that it provides a commitment to uninterrupted time for learning. Routinely planned learning blocks mean that students know what is expected of them.

"Attempts to cover too many topics too quickly may hinder learning. ... Providing students with time to learn also includes providing enough time for them to process information. ... The implication is that learning cannot be rushed; the complex cognitive activity of information integration requires time."

(Bransford, Brown, & Cocking, 2000, p. 58)

Uninterrupted teaching time only provides an opportunity to learn. To gain the most from a mathematics block, lessons need to have *a clear coherent theme*.

A mathematics block is rarely composed of a single activity. Rather, it is like a good story. A good story engages the reader's interest in a *series of interconnected events*, which are best understood in the context of what has happened earlier and what will follow. It is also possible to revisit the lesson theme at another time during the day, to help students make connections.

What type of mathematics lesson?

Lasting success in mathematics comes from understanding. Too often students see mathematics as a collection of steps and tricks that they must learn. This misconception contributes to common errors (Figure 3), such as lining up numbers to carry out arithmetic (procedural knowledge) without understanding place value (conceptual knowledge).

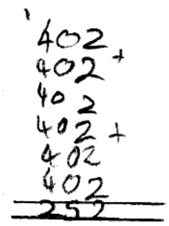

Figure 3. *Adding without understanding*

To determine what students already know and understand it is important to have them explain their reasoning. For students to make connections in mathematics, learning must build on what they understand (conceptual knowledge) and not simply their procedural knowledge.

When students are asked to explain their thinking, they are forced to organise their ideas. It is as important to expect students to explain their reasoning when they are correct, as it is when they are incorrect. In this way students learn to follow others' reasoning and have the opportunity to develop and extend their understanding.

Students communicating their reasoning in mathematics should include writing as well as talking. Students' recordings in mathematics can be pictographic or iconic (e.g. tally marks) as well as using words and the formal mathematics symbol system. When students record in class they have to revisit their thinking and reflect on their ideas. Students' recordings also give teachers a way to determine how their students are thinking and what they understand.

Presenting mathematics lessons using real world contexts can give students access to otherwise abstract mathematical ideas. Contexts can also be created from imaginary situations, and children's books can be ideal starting points for mathematics lessons.

A Year 1 mathematics lesson

After reading the book *One is a Snail, Ten is a Crab,* the teacher asked the following question: *I can see 20 feet, who could be there?* The students were encouraged to talk with a partner about this and then to draw their answers. The students were also asked to record what they had drawn (Figure 4).

Figure 4. *1 crab and 10 snails = 20*

When the students were finished, the teacher asked them to leave their drawings and to walk around the classroom to see if everyone had the same drawing. She then followed up with the question, *What else could I see?*

Multiple different correct responses are possible within the context of the story. Students can also make connections with additional information as is evident in the following student's response (Figure 5).

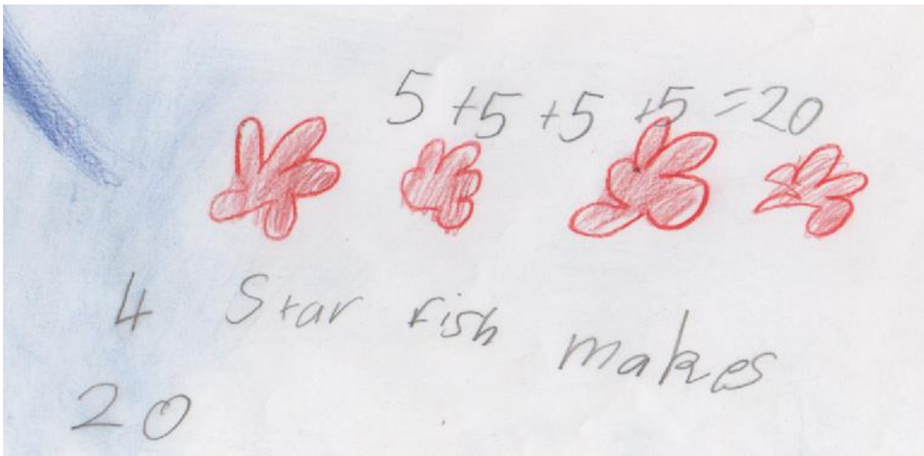

Figure 5. *4 star fish makes 20*

The teacher's intent with this lesson was to have the students recognise that different numbers can have the same total; that is, there are different ways to partition 20 (Aspect 3 of the *Numeracy Continuum K-10*).

A second goal for the lesson was to introduce a context for linking addition and subtraction questions including those involving a missing subtrahend. To achieve this goal the teacher asked the following question: *There were 23 feet, some walked away and now there are 10 left. Who walked away?*

The student's response (Figure 6) is exceptionally comprehensive in recording the problem solution pictorially and as a correct number sentence using addition. The answer is then transformed into prose before the solution is recast as a number sentence involving subtraction.

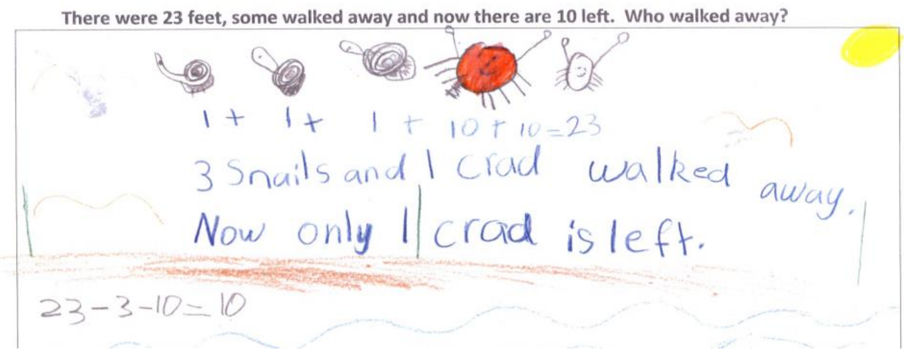

Figure 6. *Recording the solution pictorially, as an additive number sentence, in words and transformed into subtraction using a complex number sentence.*

One of the advantages of this lesson is that it is accessible to students with different levels of experience. It also readily lends itself to extension. Using the problem posed by the teacher as a scaffold, ask the students to create (and solve) their own "walked away" problems.

Summing up

- Strong early mathematics teaching is critical to the academic performance of students.
- Mathematics blocks provide a commitment to uninterrupted time for learning.
- Lasting success in mathematics comes from understanding.
- It is important to have students explain their reasoning to determine what they already understand.
- Students communicating their reasoning in mathematics should include writing as well as talking.

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Further Information

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